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# Solving SUSY GUT Problems: Gauge Hierarchy and Fermion Masses <sup>\*</sup>

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## Abstract

The supersymmetric  $SU(6)$  model accompanied by the flavour-blind discrete symmetry  $Z_3$  can successfully deal with such key problems of SUSY GUTs, as are the gauge hierarchy/doublet-triplet splitting,  $\mu$ -problem and flavour problem. The Higgs doublets arise as Goldstone modes of the spontaneously broken *accidental* global  $SU(6) \times U(6)$  symmetry of the Higgs superpotential. Their couplings to fermions have peculiarities leading to the consistent picture of the quark and lepton masses and mixing, without invoking any of horizontal symmetry/zero texture concepts. In particular, the only particle that has direct Yukawa coupling with the Higgs doublet is top quark. Other fermion masses appear from the higher order operators, with natural mass hierarchy. Specific mass formulas are also obtained.

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## 1. Introduction

The experimental developments of the last few years have confronted us with the impressive phenomenon of gauge coupling crossing [1] in the framework of minimal supersymmetric standard model (MSSM): at the scale  $M_G \simeq 10^{16}$  GeV,  $SU(3) \times SU(2) \times U(1)$  can be consistently embedded in  $SU(5)$ , which in itself can be further extended to some larger group at some larger scale. At this point two ideas, elegant GUT and beautiful SUSY, already long time wanted each other, successfully meet. It has become completely clear that GUT without SUSY is not viable [1]: non-supersymmetric  $SU(5)$  is excluded while the larger GUTs require some intermediate scale as an extra parameter, whereas the SUSY  $SU(5)$  prediction [2] for  $\sin^2 \theta_W$  well agrees with experiment. On the other hand, SUSY without GUT (i.e. MSSM directly from the string unification), gives too small  $\sin^2 \theta_W$ . In this view, it is more attractive to think that at compactification scale  $M \sim 10^{18}$  GeV, the string theory first reduces to some SUSY GUT containing  $SU(5)$ , which then breaks down to standard  $SU(3) \times SU(2) \times U(1)$  at  $M_G \simeq 10^{16}$  GeV.

Besides this experimental hint, grand unification needs supersymmetry for conceptual reasons [5], related to so-called gauge hierarchy problem. At the level of standard model this is essentially a problem of the Higgs mass ( $\sim M_W$ ) stability against radiative corrections (quadratic divergences). It is removed as soon as one appeals to SUSY, which links the masses of scalars with those of their fermion superpartners while the latter are protected by the chiral symmetry. However, in the context of grand unification the gauge hierarchy problem rather concerns the origin of scales: why the electroweak scale is so small as compared to the GUT scale  $M_G$ , which in itself is not far from the Planck scale. This question is inevitably connected with the puzzle of the so-called doublet-triplet (DT) splitting: in GUT supermultiplets the MSSM Higgs doublets are accompanied by the colour triplet partners, which mediate an unacceptably fast proton decay (especially via  $d = 5$  operators [3]) unless their masses are very large ( $\sim M_G$ ). In addition light triplets, even being decoupled from quarks and leptons by some means [4], would spoil the unification of gauge couplings.

For example, in minimal SUSY  $SU(5)$  with fermions  $(\bar{5} + 10)_i$ , where  $i = 1, 2, 3$  is a family index, and Higgs sector consisting of superfields  $\Phi(24)$  and  $\phi(5) + \bar{\phi}(\bar{5})$ , the  $SU(3) \times SU(2) \times U(1)$  decomposition of the latter is  $\phi(5) = T(3, 1) + h_2(1, 2)$  and  $\bar{\phi}(\bar{5}) = \bar{T}(\bar{3}, 1) + h_1(1, \bar{2})$ , where  $h_{1,2}$  are the MSSM higgs doublets and  $T, \bar{T}$  are their triplet partners. The only source of DT splitting can be the interaction of  $\phi, \bar{\phi}$  with the 24-plet  $\Phi$ . Indeed, the most general superpotential of these fields has the form:

$$W_{Higgs} = M\Phi^2 + m\bar{\phi}\phi + \lambda\Phi^3 + f\bar{\phi}\Phi\phi \quad (1)$$

The  $SU(5)$  breaking to  $SU(3) \times SU(2) \times U(1)$  is given by supersymmetric vacuum  $\langle \Phi \rangle = V_G \text{diag}(2, 2, 2, -3, -3)$ ,  $\langle \phi \rangle, \langle \bar{\phi} \rangle = 0$ , with  $V_G = 2M/3\lambda$ . Then the masses of the  $T$  and  $h$  fragments are respectively  $m + 2fV_G$  and  $m - 3fV_G$ , so that massless

doublets require the relation

$$3\lambda m = 2fM \quad (2)$$

and mass of triplet is unavoidably  $O(M)$  in this case. Non-renormalization theorem guarantees that in the exact supersymmetry limit this constraint is stable against radiative corrections. However, this so-called technical solution [5] is nothing but *fine tuning* of different unrelated parameters in the superpotential.

The actual task is to achieve the DT-splitting in a natural way (without fine tuning) due to symmetry reasons, which implies a certain choice of GUT symmetry and its field content. Several attempts have been done, but none of them seems satisfactory. The sliding singlet scenario [6] was shown to be unstable under radiative corrections [7]. The group-theoretical trick known as a missing partner mechanism [8] needs rather complicated Higgs sector when implemented in  $SU(5)$ . In a most economic way it works in the *flipped  $SU(5)$*  [9], which is not however a *genuine* GUT unifying the gauge couplings. The missing VEV mechanism [10] implemented in SUSY  $SO(10)$  also requires rather artificial Higgs sector if one attempts [11] to support it by some symmetry. In addition, the 'missing' (partner, VEV) mechanisms once being motivated by symmetry reasons so that the Higgs doublets are strictly massless in the exact supersymmetry limit, miss also a solution of so-called  $\mu$ -problem [12]. The  $\mu$ -term of the order of soft SUSY breaking terms, in fact should be introduced by hand.

Another theoretical weakness of SUSY GUTs is a lack in the understanding of flavour. Although GUTs can potentially unify the Yukawa couplings within one family, the origin of inter-family hierarchy and weak mixing pattern remains open. Moreover, in the light families the Yukawa unification simply contradicts to observed mass pattern, though the  $b - \tau$  Yukawa unification [13] may constitute a case of partial but significant success. In order to deal with the flavour problem in GUT frameworks, some additional ideas (horizontal symmetry, zero textures) are required [14, 15].

## 2. GIFT – Goldstones Instead of Fine Tuning

An attractive possibility to solve the gauge hierarchy problem and the related problem of the DT splitting, suggested in [16, 17, 18], can be simply phrased as follows: Higgs doublets can appear as Goldstone modes of a spontaneously broken global symmetry, which is larger than the local symmetry of the GUT. These doublets, being strictly massless in the exact SUSY limit, acquire nonzero masses after supersymmetry breaking and thereby trigger the electroweak symmetry breaking. In ref. [17] this mechanism was elegantly named as GIFT.

In refs. [16, 17] GIFT mechanism was implemented in  $SU(5)$  model, by *ad hoc* assumption that the Higgs superpotential has larger global symmetry  $SU(6)$ . This was done by adding a singlet superfield  $I$  to the minimal Higgs sector of  $SU(5)$ :

$\Phi(24) + \bar{\phi}(\bar{5}) + \phi(5) + I(1)$  is just the  $SU(5)$  decomposition of the  $SU(6)$  adjoint representation  $\Sigma(35)$ . If one assumes that the Higgs superpotential has the simple form  $W_{Higgs} = M\Sigma^2 + \lambda\Sigma^3$ , then it possesses the  $SU(6)$  global symmetry. The supersymmetric ground state  $\langle \Sigma \rangle = V_G \text{diag}(1, 1, 1, 1, -2, -2)$  (one among the other discretely degenerated vacua), breaking  $SU(6)$  down to  $SU(4) \times SU(2) \times U(1)$ , gives rise to Goldstone supermultiplets in fragments  $(4, \bar{2}) + (\bar{4}, 2)$ . At the same time the gauged part  $SU(5) \subset SU(6)$  breaks down to  $SU(3) \times SU(2) \times U(1)$ , so that the fragments  $(3, \bar{2}) + (\bar{3}, 2)$  are eaten up by Higgs mechanism. The remaining Goldstone fragments  $h_1 = (1, 2)$  and  $h_2 = (1, \bar{2})$  stay massless until supersymmetry (and thus also larger global symmetry of Higgs Potential) remains unbroken. However, the global  $SU(6)$  symmetry in the Higgs sector of  $SU(5)$  seems rather artificial. In general the Higgs superpotential of the fields involved should be

$$W_{Higgs} = \mu^2 I + M\Phi^2 + M'I^2 + m\bar{\phi}\phi + \lambda\Phi^3 + \lambda'I^3 + \lambda''\Phi^2 I + f\bar{\phi}\Phi\phi + f'I\bar{\phi}\phi \quad (3)$$

while the  $SU(6)$  invariance is equivalent to imposing the following constraints

$$\mu = 0, \quad M = M' = m/2, \quad \lambda = -\sqrt{15/8}\lambda' = \sqrt{10/3}\lambda'' = f/3 = -\sqrt{5/6}f' \quad (4)$$

Without valid dynamical or symmetry reasons these constraints look as *several* unnatural fine tunings instead of *one* tuning (2) needed in minimal SUSY  $SU(5)$ . Thus, if one remains within the SUSY  $SU(5)$  frames, GIFT<sup>1)</sup> is LOST<sup>2)</sup>.

A much more attractive scenario is that the larger global symmetry arises in an accidental way [18]. In other words, it should be an automatic consequence of the gauge symmetry and the field content of the model. Obviously, this requires extension of the  $SU(5)$  gauge symmetry, say to  $SU(6)$  [18], with the anomaly-free fermion sector consisting of chiral superfields  $(\bar{6} + \bar{6}' + 15)_i$ , where  $i = 1, 2, 3$  is a family index. The Higgs sector contains supermultiplets  $\Sigma$  and  $H + \bar{H}$ , respectively in adjoint 35 and fundamental  $6 + \bar{6}$  representations. If the Higgs superpotential has a structure  $W = M\Sigma^2 + \lambda\Sigma^3 + S(\bar{H}H - V_H^2)$ , where  $S$  is an auxiliary singlet, then it acquires an extra global symmetry  $SU(6)_\Sigma \times U(6)_H$ .<sup>3)</sup> In the exact SUSY limit the vacuum state has continuous degeneration:  $\langle H \rangle = \langle \bar{H} \rangle = V_H(1, 0, 0, 0, 0, 0)$ ,  $\langle \Sigma \rangle = V_G U^\dagger \text{diag}(1, 1, 1, 1, -2, -2)U$ , where  $U$  is arbitrary  $6 \times 6$  unitary matrix.<sup>4)</sup> If the true vacuum state corresponds to  $U = 1$  (as it can appear after SUSY breaking), then these VEVs break the  $SU(6)$  gauge symmetry down to the standard  $SU(3) \times SU(2) \times U(1)$  symmetry. At the same time, the global symmetry  $SU(6)_\Sigma \times U(6)_H$  is

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<sup>1)</sup> *Goldstones Instead of Fine Tuning*

<sup>2)</sup> *Lots Of Strange Tunings*

<sup>3)</sup> Notice, however, that the crossing term  $\bar{H}\Sigma H$  is put to zero by hand.

<sup>4)</sup> In fact,  $SU(6)_\Sigma \times SU(6)_H$  is not a global symmetry of a whole Higgs Lagrangian, but only of the Higgs superpotential. In particular, the Yukawa as well as the gauge couplings ( $D$ -terms) do not respect it. However, in the exact supersymmetry limit (i) it is effective for the field configurations on the vacuum valley, where  $D = 0$ , (ii) it cannot be spoiled by the radiative corrections from the Yukawa interactions, owing to non-renormalization theorem.

spontaneously broken to  $[SU(4) \times SU(2) \times U(1)]_\Sigma \times U(5)_H$ , and the corresponding Goldstone modes are presented by fragments  $[(4, \bar{2}) + (\bar{4}, 2)]_\Sigma \rightarrow [\bar{T}'(3, \bar{2}) + T'(\bar{3}, 2) + \bar{h}(1, \bar{2}) + h(1, 2)]_\Sigma$  and  $\bar{5}_{\bar{H}} + 5_H \rightarrow [\bar{T}(\bar{3}, 1) + \bar{h}(1, \bar{2})]_{\bar{H}} + [T(3, 1) + h(1, 2)]_H$ . Clearly, all these are eaten up by corresponding fragments in vector superfields of the gauge  $SU(6)$  symmetry, except the combinations  $h_1 \propto V_H \bar{h}_\Sigma - 3V_G \bar{h}_{\bar{H}}$ ,  $h_2 \propto V_H h_\Sigma - 3V_G h_H$ , which remain massless and can be identified with the MSSM Higgs doublets.

In order to maintain the gauge coupling unification, we have to assume  $V_H \geq V_G$ , so that  $SU(6)$  is first broken by  $H, \bar{H}$  down to  $SU(5)$ . Then, at the scale  $V_G \simeq 10^{16}$  GeV, the VEV of  $\Sigma$  breaks  $SU(5)$  down to  $SU(3) \times SU(2) \times U(1)$ . As far as the superpotential of  $\Sigma$  does not feel the local symmetry breaking by the  $H, \bar{H}$  VEVs and continues to carry the global  $SU(6)_\Sigma$  symmetry, in the limit  $V_H \gg V_G$  the theory automatically reduces to the  $SU(5)$  GIFT model of refs. [16, 17].

After the SUSY breaking enters the game (presumably through the hidden supergravity sector), the Higgs potential, in addition to the (supersymmetric) squared  $F$  and  $D$  terms, includes also the soft SUSY breaking terms [25]. These are given by  $V_{SB} = AmW_3 + BmW_2 + m^2|\phi_k|^2$ , where  $\phi_k$  imply all scalar fields involved,  $W_{3,2}$  are terms in superpotential respectively trilinear and bilinear in  $\phi_k$ , and  $A, B, m$  are soft breaking parameters. Due to these terms the VEV of  $\Sigma$  is shifted, as compared to the one calculated in the exact SUSY limit, by an amount of  $\sim m$ . Through the  $\Sigma^3$  term in the superpotential, this shift gives rise to term  $\mu h_1 h_2$ . Thus, the GIFT scenario automatically solves the  $\mu$ -problem: the (supersymmetric)  $\mu$ -term for the resulting MSSM in fact arises as a result of SUSY breaking, with  $\mu \sim m$ .

The Higgs doublets acquire also the soft SUSY breaking mass terms, but not all of them immediately. Clearly,  $V_{SB}$  also respects the larger global symmetry  $SU(6)_\Sigma \times U(6)_H$ , so that one combination of the scalars  $h_1$  and  $h_2$ , namely  $\tilde{h} = h_1 + h_2^*$  remains massless as a truly Goldstone boson. However, as far as SUSY breaking relaxes radiative corrections, the latter will remove the vacuum degeneracy and provide non-zero mass to  $\tilde{h}$  (situation, much similar to the case of axion). The effects of radiative corrections, which lift vacuum degeneracy and lead to the electroweak symmetry breaking, were studied in ref. [19]. It was shown that GIFT scenario does not imply any upper bound on the top quark mass, in spite of earlier claims [17, 20] and it can go up to its infrared fixed limit.

In fact, the  $SU(6)$  model [18] is a minimal extension of the standard  $SU(5)$  model. At the scale  $V_H$  the fermion content is reduced to the minimal fermion content of standard  $SU(5)$ . Indeed, the  $SU(5)$  decomposition of various supermultiplets reads

$$\begin{aligned} H &= (5 + 1)_H, & \bar{H} &= (\bar{5} + 1)_{\bar{H}}, & \Sigma &= (24 + 5 + \bar{5} + 1)_\Sigma \\ \bar{6}_i &= (\bar{5} + 1)_i, & \bar{6}'_i &= (\bar{5} + 1)'_i, & 15_i &= (10 + 5)_i \quad (i = 1, 2, 3) \end{aligned} \quad (5)$$

so that the fermion sector at this scale consists of six  $\bar{5}$ -plets, three 5-plets, three 10-plets and six singlets. According to the survival hypothesis [21], after the breaking  $SU(6) \rightarrow SU(5)$  the extra fermions  $(5 + \bar{5}')_i$  become heavy (with masses  $\sim V_H$ ) owing to the Yukawa couplings  $\Gamma_{ij} 15_i \bar{H} \bar{6}'_j$ , and decouple from the light states

which remain in  $(\bar{5} + 10)_i$  and singlets. Thus, below the scale  $V_H$  we are left with minimal  $SU(5)$  GUT with three standard fermion families.

The  $SU(6)$  theory [18] drastically differs from any other GUT approaches. Usually, in GUTs the Higgs sector consists of two different sets: one is for the GUT symmetry breaking (e.g. 24-plet in  $SU(5)$ ), while another is just for the electroweak symmetry breaking and fermion mass generation (like  $5 + \bar{5}$  in  $SU(5)$ ). In contrast, in the  $SU(6)$  theory no special superfields are introduced for the second function. The Higgs sector consisting of the 35-plet and  $6 + \bar{6}$ , is a minimal one needed for the  $SU(6)$  breaking down to the  $SU(3) \times SU(2) \times U(1)$ . As for the Higgs doublets, they arise as Goldstone modes, from the  $SU(2) \times U(1)$  doublet fragments in  $\Sigma$  and  $H, \bar{H}$ .

Due to this reason, their couplings with the fermion fields have some peculiarities, which could provide new possibilities towards the understanding of flavour. Namely, if by chance the Yukawa superpotential also respects the  $SU(6)_\Sigma \times U(6)_H$  global symmetry, then the Higgs doublets, as Goldstone fields, have *vanishing* Yukawa couplings with the fermions that remain massless after the  $SU(6)$  symmetry breaking. In fact, these are the chiral families  $(\bar{5} + 10)_i$  of ordinary quarks and leptons. Yukawa couplings  $15 \bar{H} \bar{6}$  respect extra global symmetry and cannot generate their masses, so that one has to invoke the higher order operators scaled by inverse powers of some large mass  $M \gg V_H$ . These could appear due to nonperturbative quantum gravity effects, with  $M \sim M_{Pl}$ . Alternatively, they can arise by integrating out some heavy states with masses above the  $SU(6)$  breaking scale. Indeed, such states in vectorlike (real) representations can naturally present in SUSY (stringy) GUTs. According to survival hypothesis, they should acquire maximal allowed masses  $M$  if there are no symmetry reasons to keep them light. Then masses of ordinary light fermions can appear as a result of 'seesaw' mixing with these heavy states [22]. In model [18], operators relevant for down quarks appear as  $\frac{1}{M} 15 \Sigma \bar{H} \bar{6}$  while the operators relevant for upper quarks are  $\frac{1}{M^2} 15 H \Sigma H 15$ . So, it seems that model leads to unacceptable case  $m_b \gg m_t$ .

As it was shown in ref. [23], the problem can be resolved by introducing a fermion 20-plet, which  $SU(5)$  content is  $20 = 10 + \bar{10}$ . Since 20 is a pseudo-real representation (the tensor product  $20 \times 20$  contains singlet only in an antisymmetric combination), the survival hypothesis does not apply to it. More generally, if in original theory 20-plets present in odd number then *one* of them should inevitably stay massless. Then its Yukawa couplings  $g_{20} \Sigma 20$  and  $G_i 20 H 15_i$  *explicitly* violate the global  $SU(6)_\Sigma \times U(6)_H$  symmetry. As a result, the *only* particle which gets direct Yukawa coupling with the 'Goldstone' Higgs doublet is an upper quark contained in 20, that is top quark. Other fermions stay massless at this level, and for generating their masses one has to appeal to higher order operators. In order to achieve a proper operator structure, additional symmetries are needed. On the other hand, consistency of the GIFT scenario also requires some extra symmetry in order to forbid the crossing term  $\bar{H} \Sigma H$  – otherwise the Higgs superpotential has no accidental

global symmetry.

Below we describe a consistent SUSY  $SU(6)$  model with flavour-blind discrete  $Z_3$  symmetry [24]. The role of the latter is important: first, it guarantees that Higgs superpotential has automatic global  $SU(6)_\Sigma \times U(6)_H$  symmetry without putting to zero of some terms by hand, and second, it provides proper structure of the higher order operators generating realistic mass and mixing pattern for *all* fermion families.

## 2. $SU(6) \times Z_3$ model

Consider the supersymmetric model with  $SU(6)$  gauge symmetry, with the set of chiral superfields consisting of two sectors:

- (i) The ‘Higgs’ sector: vectorlike supermultiplets  $\Sigma_1(35)$ ,  $\Sigma_2(35)$ ,  $H(6)$ ,  $\bar{H}(\bar{6})$  and an auxiliary singlet  $S$ .
- (ii) The ‘fermion’ sector: chiral, anomaly free supermultiplets  $(\bar{6} + \bar{6}')_i$ ,  $15_i$  ( $i = 1, 2, 3$  is a family index) and  $20$ .

We introduce also two flavour-blind discrete symmetries. One is usual matter parity  $Z_2$ , under which the fermion superfields change the sign while the Higgs ones stay invariant. Such a matter parity, equivalent to R parity, ensures the proton stability. Another discrete symmetry is  $Z_3$  acting in the following way:

$$\begin{aligned} \Sigma_1 &\rightarrow \omega \Sigma_1, & \Sigma_2 &\rightarrow \bar{\omega} \Sigma_2, & H, \bar{H} &\rightarrow H, \bar{H}, & S &\rightarrow S \\ \bar{6}_{1,2}^i &\rightarrow \omega \bar{6}_{1,2}^i, & 15^i &\rightarrow \bar{\omega} 15^i, & 20 &\rightarrow \omega 20 & (\omega = e^{i\frac{2\pi}{3}}) \end{aligned} \quad (6)$$

Let us consider first the Higgs sector. The most general renormalizable superpotential compatible with the  $SU(6) \times Z_3$  symmetry is  $W_{Higgs} = W_\Sigma + W_H + W(S)$ , where  $W(S)$  is a polynomial containing linear, quadratic and cubic terms, and

$$W_\Sigma = M \Sigma_1 \Sigma_2 + \lambda_0 S \Sigma_1 \Sigma_2 + \lambda_1 \Sigma_1^3 + \lambda_2 \Sigma_2^3, \quad W_H = M' \bar{H} H + \lambda S \bar{H} H \quad (7)$$

This superpotential automatically has larger global symmetry  $SU(6)_\Sigma \times U(6)_H$ , related to independent transformations of the  $\Sigma$  and  $H$  fields. In the exact supersymmetry limit, the condition of vanishing  $F$  and  $D$  terms allows, among other discretely (and continuously) degenerated vacua, the VEV configuration

$$\begin{aligned} \langle \Sigma_{1,2} \rangle &= V_{1,2} \text{ diag}(1, 1, 1, 1, -2, -2) \\ \langle H \rangle &= \langle \bar{H} \rangle = V_H (1, 0, 0, 0, 0, 0) \end{aligned} \quad (8)$$

For a proper parameter range, these configuration can appear as a true vacuum state after that the vacuum degeneracy is lifted by soft SUSY breaking and subsequent radiative corrections [19].

The VEVs (8) lead to the needed pattern of gauge symmetry breaking:  $H, \bar{H}$  break  $SU(6)$  down to  $SU(5)$ , while  $\Sigma_{1,2}$  break  $SU(6)$  down to  $SU(4) \times SU(2) \times U(1)$ .

Both channels together break the local symmetry down to  $SU(3) \times SU(2) \times U(1)$ . At the same time, the global symmetry  $SU(6)_\Sigma \times U(6)_H$  is broken down to  $[SU(4) \times SU(2) \times U(1)]_\Sigma \times U(5)_H$ . The Goldstone degrees which survive from being eaten by Higgs mechanism constitute the couple  $h_1 + h_2$  of the MSSM Higgs doublets, which in terms of the doublet (anti-doublet) fragments in  $\Sigma_{1,2}$  and  $H, \bar{H}$  are given as

$$\begin{aligned} h_2 &= \cos \alpha (\cos \gamma h_{\Sigma_1} + \sin \gamma h_{\Sigma_2}) + \sin \alpha h_H \\ h_1 &= \cos \alpha (\cos \gamma \bar{h}_{\Sigma_1} + \sin \gamma \bar{h}_{\Sigma_2}) + \sin \alpha \bar{h}_{\bar{H}} \end{aligned} \quad (9)$$

where  $\tan \gamma = V_2/V_1$  and  $\tan \alpha = 3V_G/V_H$ ,  $V_G = (V_1^2 + V_2^2)^{1/2}$ . In the following we adopt the case  $V_H \gg V_1 \gg V_2$ . Thus, in this case the Higgs doublets dominantly come from  $\Sigma_1$ , while in  $\Sigma_2$  and  $H, \bar{H}$  they are contained with small weights  $\varepsilon_2/\varepsilon_1$  and  $3\varepsilon_1$  respectively, where  $\varepsilon_{1,2} = V_{1,2}/V_H$ .

#### 4. Fermion masses

The most general Yukawa superpotential compatible with the  $SU(6) \times Z_3$  symmetry has the form

$$W_{Yuk} = g 20\Sigma_1 20 + G 20H 15_3 + \Gamma_{ij} 15_i \bar{H} \bar{6}'_j \quad i, j = 1, 2, 3 \quad (10)$$

where all Yukawa coupling constants are assumed to be  $O(1)$  (for comparison, we remind that the gauge coupling constant at GUT scale is  $g_{GUT} \simeq 0.7$ ). Without loss of generality, one can always redefine the basis in 15-plets so that only the  $15_3$  state couples to 20-plet in (10). Also, among six  $\bar{6}$ -plets one can always choose the basis when only three of them (denoted in eq. (10) as  $\bar{6}'_{1,2,3}$ ) couple to  $15_{1,2,3}$ , while other three states  $\bar{6}_{1,2,3}$  have no Yukawa couplings.

For  $V_H \gg V_G$ , already at the breaking  $SU(6) \rightarrow SU(5)$ , the light fermion states are identified from this superpotential, whereas the extra fermion states become superheavy. Indeed, the  $SU(5) \supset SU(3) \times SU(2) \times U(1)$  decomposition of the fermion multiplets under consideration reads

$$\begin{aligned} 20 &= 10 + \bar{10} = (q + u^c + e^c)_{10} + (Q^c + U + E)_{\bar{10}} \\ 15_i &= (10 + 5)_i = (q_i + u_i^c + e_i^c)_{10} + (D_i + L_i^c)_5 \\ \bar{6}_i &= (\bar{5} + 1)_i = (d_i^c + l_i)_{\bar{5}} + N_i \\ \bar{6}'_i &= (\bar{5} + 1)'_i = (D_i^c + L_i)_{\bar{5}'} + N'_i \end{aligned} \quad (11)$$

According to eq. (10), the extra fermion pieces with non-standard  $SU(5)$  content, namely  $\bar{10}$  and  $5_{1,2,3}$ , form massive particles being coupled with  $10_3$  and  $\bar{5}'_{1,2,3}$

$$G V_H \bar{10} 10_3 + \Gamma_{ij} V_H 5_i \bar{5}'_j + g V_1 (U u^c - 2E e^c) \quad (12)$$

and thereby decouple from the light states which remain in  $\bar{5}_{1,2,3}$ ,  $10_{1,2}$  and  $10$  (if neglect the small,  $\sim \varepsilon_1$  mixing between the  $u^c - u_3^c$  and  $e^c - e_3^c$  states). On the other hand, since the couplings of 20-plet explicitly violate the global  $SU(6)_\Sigma \times U(6)_H$  symmetry, the Higgs doublet  $h_2$  has *non-vanishing* couplings with up-type quarks from 20- and  $15_3$ -plets. Indeed, it follows from eq. (10) that *only* Yukawa coupling relevant for light states is contained in  $g20\Sigma_120 \rightarrow g105_{\Sigma_1}10$ . Therefore, only one up-type quark (to be identified with top quark), dominantly contained in 20-plet, has a direct Yukawa coupling  $g qu^c h_2$ , so that its mass has to be in the 100 GeV range. Other fermions stay massless at this level, unless we invoke the higher order operators explicitly violating the accidental global symmetry.

Higher order operators scaled by inverse powers of some large mass  $M \gg V_H$  could appear due to quantum gravity effects, with  $M \sim M_{Pl}$ . Alternatively, they can arise by integrating out some heavy states with masses  $M \gg V_H$ . In the Sect. 5 we adopt the second viewpoint, namely that these operators appear from the exchange of some heavy fermion superfields [22]. The reason is twofold: first, as we see shortly, the fermion mass pattern favours the scale  $M \sim 10^{18}$  GeV (string scale?) rather than  $M_{Pl}$ , and second, the mechanism of heavy fermion exchange is rather instructive for obtaining the realistic fermion mass pattern.

Before addressing the concrete scheme of heavy fermion exchanges, let us start with the general operator analysis.  $Z_3$  symmetry forbids any ‘Yukawa’ operator in the superpotential at  $1/M$  order.<sup>5)</sup> However, operators at the next ( $1/M^2$ ) order are allowed and they are the following:

$$\mathcal{A} = \frac{a}{M^2} 20\bar{H}\Sigma_1\bar{H}\bar{6}_3, \quad \mathcal{B} = \frac{b_{ij}}{M^2} 15_i H \Sigma_2 H 15_j, \quad \mathcal{C} = \frac{c_{ik}}{M^2} 15_i (\Sigma_1 \Sigma_2 \bar{H}) \bar{6}_k \quad (13)$$

where  $a, b, c$  are  $O(1)$  ‘Yukawa’ constants. Analogous operators involving heavy  $\bar{6}'_i$  states are irrelevant for the light fermion masses. According to eq. (12) the state  $10_3 \subset 15_3$  is also heavy, and it is decoupled from the light particle spectrum. Therefore, operators (13) are relevant only for  $10 \subset 20$ ,  $10_i \subset 15_i$  ( $i = 1, 2$ ) and  $\bar{5}_k \subset \bar{6}_k$  ( $k = 1, 2, 3$ ) states. Without loss of generality, we redefine the basis of  $\bar{6}$ -plets so that only the  $\bar{6}_3$  state couples to 20-plet in eq. (13). It is worth to note that in fact  $\mathcal{C}$  can contain two relevant combinations with different convolutions of the  $SU(6)$  indices indicated by brackets in an obvious way:

$$C_1 = 15(\Sigma_1 \Sigma_2 \bar{H}) \bar{6}, \quad C_2 = 15(\Sigma_1 \bar{H})(\Sigma_2 \bar{6}), \quad C_3 = (15\bar{H})(\Sigma_1 \Sigma_2 \bar{6}), \quad C_4 = (15\bar{H}\bar{6})(\Sigma_1 \Sigma_2) \quad (14)$$

$C_1$  and  $C_2$  provide different Clebsch coefficients for the down quark and charged lepton mass terms, while  $C_3$  and  $C_4$  are irrelevant for the mass generation and they lead only to some minor rotation of the heavy fermion states.

Let us analyse now the impact of these operators on the fermion masses. Obviously, the operator  $\mathcal{A}$  is responsible for the  $b$  quark and  $\tau$  lepton masses, and at the

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<sup>5)</sup>Operators involving an odd number of fermion superfields are forbidden by matter parity.

MSSM level it reduces to Yukawa couplings  $a\varepsilon_H^2 (qd_3^c + e^c l_3) h_1$ , where  $\varepsilon_H = V_H/M$ . Hence, though  $b$  and  $\tau$  belong to the same family as  $t$  (namely, to 20-plet), their Yukawa couplings are substantially (by factor  $\sim \varepsilon_H^2$ ) smaller than  $\lambda_t \approx g$ . Moreover, we automatically have almost precise  $b - \tau$  Yukawa unification at the GUT scale:

$$\lambda_b = a\varepsilon_H^2 = \lambda_\tau [1 + (\varepsilon_1 g/G)^2] \quad (15)$$

where the  $\sim \varepsilon_1^2$  correction is due to mixing between the  $e^c$  and  $e_3^c$  states (see eq. (12)).

As far as the fermions of the third family are already defined as the states belonging to  $10 \subset 20$  and  $\bar{5}_3 \subset \bar{6}_3$ , the operators  $\mathcal{B}$  and  $\mathcal{C}$  generate mass terms for the fermions of the first two families, which in general case are expected to be of the same order. In order to achieve mass splitting between the second and first families, one can assume that the ‘Yukawa’ matrices  $b_{ij}$  and  $c_{ik}$  are *rank-1* matrices, so that each of operators  $\mathcal{B}$  and  $\mathcal{C}$  can provide only one non-zero mass eigenvalue (i.e.  $c$  and  $s$  quark masses). Then, without loss of generality, we can redefine the basis of  $15_{1,2}$  and  $\bar{6}_{1,2}$  states so that these matrices have the form

$$\begin{aligned} b_{ij} &= (0, \beta)^T \bullet (0, \beta) = \begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix} \\ c_{ik} &= (\gamma_1, \gamma_2)^T \bullet (0, \delta_2, \delta_3) = \begin{pmatrix} 0 & c_2 \sin \theta & c_3 \sin \theta \\ 0 & c_2 \cos \theta & c_3 \cos \theta \end{pmatrix} \end{aligned} \quad (16)$$

where  $\tan \theta = \gamma_1/\gamma_2$ . Hence, in this basis only  $b_{22} = b$  component of the symmetric matrix  $b_{ij}$  is nonzero, and  $c$  quark should be identified as an up-quark state from  $q_2, u_2^c \subset 10_2 \subset 15_2$ . Then  $s$  quark state is the down quark state in  $q'_2 \subset 10'_2 \subset 15'_2$  and  $d_2^c \subset \bar{5}_2 \subset \bar{6}_2$ , where  $15'_2 = \sin \theta \cdot 15_1 + \cos \theta \cdot 15_2$  is an effective combination of the  $15_{1,2}$  states which couples  $\bar{6}_2$  and  $\bar{6}_3$  states (it is not difficult to recognize that in fact  $\theta$  is the Cabibbo angle, which in general tends to be  $O(1)$ ). Clearly,  $\mu$ -lepton is also contained in  $15'_2 + \bar{6}_2$ . Thus, operators  $\mathcal{B}$  and  $\mathcal{C}$  reduce to MSSM Yukawa couplings for the second family quarks and leptons

$$b(\varepsilon_2/\varepsilon_1)\varepsilon_H^2 q_2 u_2^c h_2, \quad c_{2,3}\varepsilon_2\varepsilon_H^2 (q'_2 d_{2,3}^c + K e_2' l_{2,3}) h_1 \quad (17)$$

where  $K$  is the Clebsch coefficient depending on weights of operators (14) entering  $\mathcal{C}$  (for example,  $K = 1$  if  $\mathcal{C} \propto C_1$  and  $K = -2$  if  $\mathcal{C} \propto C_2$ ).

For the first family fermion masses one can appeal to  $1/M^3$  operators which can be presented as following:

$$\mathcal{D} = \frac{d_{ik}}{M^3} 15'_i \Sigma_1^3 \bar{H} \bar{6}_k, \quad \mathcal{E} = \frac{e_{ij}}{M^3} 15_i H \Sigma_1^2 H 15_j \quad (18)$$

where  $15'_1$  is a state orthogonal to  $15'_2$ . As in the case of operator  $\mathcal{C}$ , these operators can have different convolutions of the  $SU(6)$  indices. For  $\mathcal{D}$  the relevant combinations, giving different relative Clebsch factors  $P$  for the  $d$  quark and electron masses,

are

$$D_1 = 15(\Sigma_1^3 \bar{H})\bar{6}, \quad D_2 = 15(\Sigma_1^2 \bar{H})(\Sigma_1 \bar{6}), \quad D_3 = 15(\Sigma_1 \bar{H})(\Sigma_1^2 \bar{6}), \quad D_4 = 15(\Sigma_1 \bar{H})\bar{6}(\Sigma_1^2) \quad (19)$$

Operator  $\mathcal{D}$ , with arbitrary couplings  $d_{ij}$  of the order of 1, will provide  $d$ -quark and electron masses in the correct range. As for the operator  $\mathcal{E}$ , for  $e_{11} \sim 1$ , it leads to somewhat large value of  $u$ -quark mass. Therefore, it is more suggestive to think that only  $e_{12}$  couplings are non-zero, while  $e_{11} = 0$ . As we show in Sect. 5, this is really the case in the context of heavy fermion exchange model.

Keeping only the leading contributions to each component, for the quark and lepton Yukawa couplings at the GUT scale we obtain

$$\begin{matrix} u_1^c & u_2^c & u^c \\ q_1 \begin{pmatrix} 0 & e_{12}\varepsilon_1\varepsilon_H^3 & 0 \\ e_{12}\varepsilon_1\varepsilon_H^3 & b(\varepsilon_2/\varepsilon_1)\varepsilon_H^2 & 0 \\ 0 & 0 & g \end{pmatrix} \cdot h_2 \\ q_2 \\ q \end{matrix} \quad (20a)$$

$$\begin{matrix} d_1^c & d_2^c & d_3^c \\ q'_1 \begin{pmatrix} d_{11}\varepsilon_1^2\varepsilon_H^3 & d_{12}\varepsilon_1^2\varepsilon_H^3 & d_{13}\varepsilon_1^2\varepsilon_H^3 \\ d_{21}\varepsilon_1^2\varepsilon_H^3 & c_2\varepsilon_2\varepsilon_H^2 & c_3\varepsilon_2\varepsilon_H^2 \\ 0 & 0 & a\varepsilon_H^2 \end{pmatrix} \cdot h_1 \\ q'_2 \\ q \end{matrix} \quad (20b)$$

$$\begin{matrix} l_1 & l_2 & l_3 \\ e'_1^c \begin{pmatrix} Pd_{11}\varepsilon_1^2\varepsilon_H^3 & Pd_{12}\varepsilon_1^2\varepsilon_H^3 & Pd_{13}\varepsilon_1^2\varepsilon_H^3 \\ Pd_{21}\varepsilon_1^2\varepsilon_H^3 & Kc_2\varepsilon_2\varepsilon_H^2 & Kc_3\varepsilon_2\varepsilon_H^2 \\ 0 & 0 & a\varepsilon_H^2 \end{pmatrix} \cdot h_1 \\ e'_2^c \\ e^c \end{matrix} \quad (20c)$$

Thus, the Yukawa coupling eigenvalues at the GUT scale are  $\lambda_t = g \sim 1$  and

$$\begin{aligned} \lambda_b = \lambda_\tau = a\varepsilon_H^2 &\Rightarrow \lambda_b/\lambda_t \sim \varepsilon_H^2 \\ \lambda_c = b(\varepsilon_2/\varepsilon_1)\varepsilon_H^2 &\Rightarrow \lambda_c/\lambda_b \sim \frac{\varepsilon_2}{\varepsilon_1} \\ \lambda_s = \lambda_\mu/K = c_2\varepsilon_2\varepsilon_H^2 &\Rightarrow \lambda_s/\lambda_b \sim \varepsilon_2 \\ \lambda_d = \lambda_e/P = d_{11}\varepsilon_1^2\varepsilon_H^3 &\Rightarrow \lambda_d/\lambda_s \sim \varepsilon_2 \left( \frac{\varepsilon_1}{\varepsilon_2} \sqrt{\varepsilon_H} \right)^2 \\ \lambda_u = \frac{e_{12}^2}{b}(\varepsilon_1^3/\varepsilon_2)\varepsilon_H^4 &\Rightarrow \lambda_u/\lambda_d \sim \frac{\varepsilon_2}{\varepsilon_1} \left( \frac{\varepsilon_1}{\varepsilon_2} \sqrt{\varepsilon_H} \right)^2 \end{aligned} \quad (21)$$

In order to connect these Yukawa constants to the physical masses of the quarks and leptons, the renormalization group (RG) running has to be considered [15, 26]. We have:

$$\begin{aligned} m_t &= \lambda_t A_u \eta_t y^6 v \sin \beta, & m_b &= \lambda_b A_d \eta_b y v \cos \beta, & m_\tau &= \lambda_\tau A_e \eta_\tau v \cos \beta \\ m_c &= \lambda_c A_u \eta_c y^3 v \sin \beta, & m_s &= \lambda_s A_d \eta_d v \cos \beta, & m_\mu &= \lambda_\mu A_e \eta_l v \cos \beta \\ m_u &= \lambda_u A_u \eta_u y^3 v \sin \beta, & m_d &= \lambda_d A_d \eta_d v \cos \beta, & m_e &= \lambda_e A_e \eta_l v \cos \beta \end{aligned} \quad (22)$$

where  $v = 174$  GeV and  $\tan \beta = v_2/v_1$  is a famous ratio of the  $h_2$  and  $h_1$  VEVs. The factors  $A_f$  account for the running induced by gauge couplings from the GUT scale  $M_G$  to the SUSY breaking scale  $M_S$  (for the definiteness we take  $M_S \simeq m_t$ ), and  $y$  includes the running induced by the top quark Yukawa coupling:

$$y = \exp \left[ -\frac{1}{16\pi^2} \int_{\ln M_S}^{\ln M_G} \lambda_t^2(\mu) d(\ln \mu) \right] \quad (23)$$

The factors  $\eta_f$  encapsulate the running from  $M_S$  down to  $m_f$  for heavy quarks  $f = t, b, c$  or  $\mu = 1$  GeV for light quarks  $f = u, d, s$ . Taking all these into the account, we see that qualitatively correct description of *all* fermion masses can be achieved with  $\varepsilon_H = V_H/M \sim 0.1$ ,  $\varepsilon_1 = V_1/V_H \sim 0.1$  and  $\varepsilon_2/\varepsilon_1 = V_2/V_1 \sim 0.3$  (so that  $\frac{\varepsilon_1}{\varepsilon_2} \sqrt{\varepsilon_H} \sim 1$ ), provided that  $\tan \beta = 1 - 1.5$ . Interestingly, this region of  $\tan \beta$  is favoured by the electroweak symmetry radiative breaking picture in the presence of  $b - \tau$  Yukawa unification (see eq. (15)). In addition,  $b - \tau$  unification and small  $\tan \beta$  require substantially large  $\lambda_t$ , actually close to its infrared fixed point [27], which implies for the physical top mass  $M_t \approx \sin \beta (190 - 210)$  GeV. As far as the scale  $V_1 \simeq 10^{16}$  GeV is fixed by the  $SU(5)$  unification of gauge couplings, these relations in turn imply that  $V_H \sim 10^{17}$  GeV and  $M \sim 10^{18}$  GeV.

Obtained mass matrices give rise to a qualitatively correct picture of quark mixing. In particular, one obtains the CKM matrix at the unification scale as

$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & s_{12} & s_{12}s_{13} - s_{13}e^{-i\delta} \\ -s_{12} & 1 & s_{23} + s_{12}s_{13}e^{-i\delta} \\ s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix} \quad (24)$$

where  $\delta$  is a CP-phase and

$$\begin{aligned} s_{12} &\approx \gamma_1/\gamma_2 \sim 1 & (|V_{us}| = 0.220 \pm 0.002) \\ s_{23} &\approx c_3 \lambda_s / c_2 \lambda_b \sim \varepsilon_2 & (|V_{cb}| = 0.04 \pm 0.01) \\ s_{13} &\approx d_{13} \lambda_d / d_{11} \lambda_b \sim \varepsilon_2^2 & (|V_{td}| = 0.03 - 0.1) \end{aligned} \quad (25)$$

for comparison, in the brackets the 'experimental' values of mixing angles are shown. The questions of the neutrino mass pattern and the proton decay features due to  $d = 5$  Higgsino mediated operators are considered in refs. [23, 24].

## 5. Yukawa couplings generated by heavy particle exchanges

From the previous section, we are left with two problems: the difficulty in splitting the masses of the first two families (in Sect. 4 the form (16) for the coupling constants in operators  $\mathcal{B}$  and  $\mathcal{C}$  was assumed by hand), and the need to suppress the coupling  $e_{11}$  in operator  $\mathcal{E}$ , which leads to unacceptably large  $u$  quark mass.

$Z_3:$	Higgs	fermions	$F$ -fermions
$\omega$	$\Sigma_1$	$\bar{6}_i, \bar{6}'_i, 20$	$15_F, \bar{15}_F^{1,2}, 20_F, 84_F$
$\bar{\omega}$	$\Sigma_2$	$15_i$	$\bar{15}_F, 15_F^{1,2}, \bar{20}_F, \bar{84}_F$
$inv.$	$S, H, \bar{H}$	—	$\bar{15}_F, 15_F^{1,2}, 20_F^{1,2}, \bar{105}_F, 105_F, \bar{210}_F, 210_F$

Table 1:  $Z_3$ -transformations of various supermultiplets.

Here we show how both problems can be solved, still without appealing to any flavour symmetry, by assuming that all higher order operators are generated by the exchanges of some heavy superfields with  $\sim M$  masses. As we see below, this mechanism provides also specific predictions for the Clebsch coefficients  $K$  and  $P$  distinguishing down quark and charged lepton masses.

Let us introduce the set of heavy vectorlike fermions (in the following referred as  $F$ -fermions) with masses  $O(M)$  and transformation properties under  $SU(6) \times Z_3$  given in Table 1. Certainly, we prescribe negative matter parity to all of them.

The operators  $\mathcal{A}, \mathcal{B}, \mathcal{C}$  obtained by the proper exchange chains are shown in Fig. 1. We see that these operators generate only the third and second family masses. Indeed, coupling  $15_F \Sigma_1 \bar{15}_F^1$  defines  $\bar{15}_F^1$  state while the corresponding  $15_F^1$  state defines  $\bar{6}_3$  through the coupling  $15_F^1 \bar{H} \bar{6}_3$ . The operator  $\mathcal{A}$  is unambiguously built in this way. On the other hand, coupling  $15_2 H 20_F$  defines  $15_2$  state, so that the operator  $\mathcal{B}$  contributes only the  $c$  quark mass. The coupling  $(\gamma_1 15_1 + \gamma_2 15_2) \Sigma_1 \bar{15}_F'$  defines  $15_2'$  state, which in general does not coincide with  $15_2$ , and the coupling  $15_F^2 \bar{H} \bar{6}_2$  defines  $\bar{6}_2$  state. Therefore, the operator  $\mathcal{C}$  providing only the  $s$  quark and  $\mu$ -lepton masses, in general implies the large Cabibbo angle,  $\tan \theta = \gamma_1/\gamma_2$ . In addition, the operator  $\mathcal{C}$  derived in this way, acts as combination  $\mathcal{C} \propto C_1 + 2C_2$  of operators (14), which leads to specific Clebsch coefficient  $K = -5$  in eq. (17).

Exchanges generating operators  $\mathcal{D}, \mathcal{E}$  are shown in Fig. 2. In reproducing these operators, we have taken into account the following restriction:  $\mathcal{D}$  built by  $F$ -fermion exchange should be irreducible to lower ( $1/M^2$ ) order operator, in order to guarantee the mass hierarchy between first and second families. In other words, the exchange chain should not allow to replace  $\Sigma_1 \Sigma_1$  by  $\Sigma_2$ . This condition requires large representations of  $SU(6)$  involved into the exchange. Then the operator  $\mathcal{D}$  built as shown in Fig. 1D acts in combination  $\mathcal{D} \propto D_1 + D_3 - D_4$ , which gives relative Clebsch coefficient  $P = 5/8$  for the  $d$  quark and electron masses. On the other hand, the operator  $\mathcal{E}$  built as in Fig. 2E, can only mix  $15_1$  state containing  $u$  quark, with  $15_2$  state containing  $c$  quark, but cannot provide direct mass term for the former.

As a result, the higher order operators obtained by the exchange of  $F$ -fermions given in Table 1, consistently reproduce the mass matrix ansatz given in Sect. 4. Moreover, specific Clebsch coefficients are obtained, leading to relations  $\lambda_d = \frac{1}{5} \lambda_\mu$

and  $\lambda_d = \frac{8}{5}\lambda_e$  (small ( $\sim \varepsilon_1$ ) corrections to these can arise from the interference of the operators  $\mathcal{C}$  and  $\mathcal{D}$ ). According to eqs. (22), these relations imply

$$\frac{m_d}{m_s} \simeq 8 \frac{m_e}{m_\mu} \approx \frac{1}{25} \quad (26)$$

In addition, by taking into account the uncertainties of renormalization factors (22), mainly due to uncertainty in  $\alpha_3(M_Z)$ , for the quark running masses at  $\mu = 1 \text{ GeV}$  we obtain

$$m_s = 90 - 150 \text{ MeV}, \quad m_d = 4 - 7 \text{ MeV} \quad (27)$$

in agreement with the experimental values.

Let us conclude with following remark. As we have seen, the fermion mass pattern requires that scales  $M$ ,  $V_H$  and  $V_G$  are related as  $V_G/V_H \sim V_H/M \sim 0.1$ . The superpotential (7) includes mass parameters, which are not related to  $M$ . Therefore, it cannot explain why the scales should be arranged in this way. Bearing in mind the possibility that considered  $SU(6)$  theory could be a stringy SUSY GUT, one can assume that the superfields  $H, \bar{H}$  and  $\Sigma_{1,2}$  are zero modes, and their superpotential has the form not containing mass terms:

$$W = S[\bar{H}H - (\varepsilon_H M)^2] + \lambda_1 \Sigma_1^3 + \lambda_2 \Sigma_2^3 + \frac{(\bar{H}H)}{M} (\Sigma_1 \Sigma_2) \quad (28)$$

The last term can be effectively obtained by exchange of the singlet superfield  $Z$  with a large mass term  $MZ^2$ , as shown in Fig. 3. Then the relation  $V_G/V_H \sim V_H/M = \varepsilon_H$  follows naturally. Certainly, the origin of small linear term ( $\varepsilon_H \sim 0.1$ ) in (28) remains unclear. It may arise due to some hidden sector outside the GUT.

Non-perturbative effects in principle could induce the higher order operators scaled by inverse powers of the Planck mass. If all such operators unavoidably occur, this would spoil the GIFT picture. For example, already the operator  $\frac{1}{M_{Pl}}(\bar{H}\Sigma_1)(\Sigma_2H)$  would provide an unacceptably large ( $\sim M_G^2/M_{Pl}$ ) mass to the Higgs doublets. One may hope, however, that not all possible structures appear in higher order terms. Alternatively, one could try to suppress dangerous high order operators by symmetry reasons, in order to achieve a consistent 'all order' solution [28].

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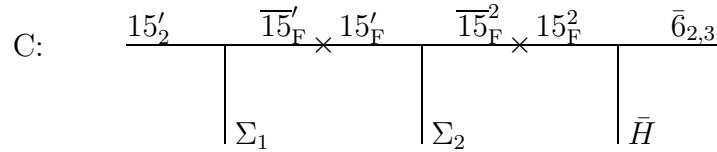
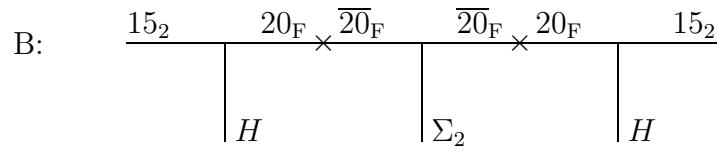
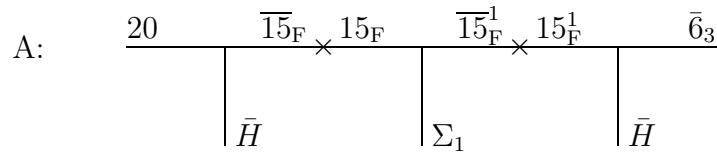


Figure 1: diagrams giving rise to the operators  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\mathcal{C}$  respectively.

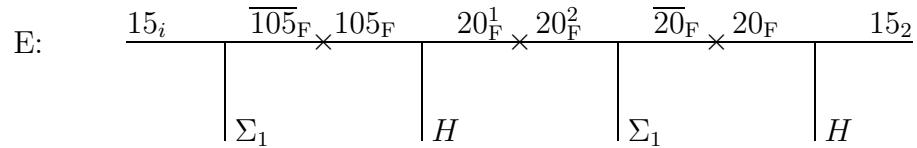
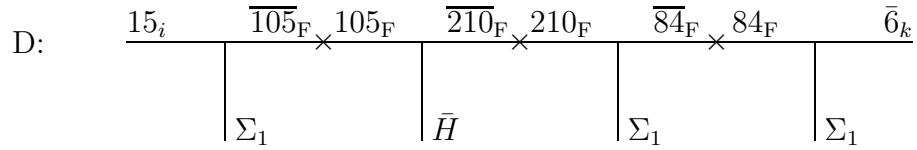


Figure 2: diagrams giving rise to the operators  $\mathcal{D}$  and  $\mathcal{E}$  respectively.

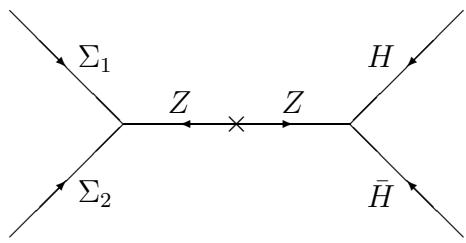


Figure 3: Diagram generating the operator  $\frac{1}{M}(\bar{H}H)(\Sigma_1\Sigma_2)$ .